REDUCING EMI OF COMMUTATOR MOTORS BY OPTIMIZING BRUSH-TO-SEGMENT WIDTH RATIO

[1] France Pavlovčič, [2] Janez Nastran

[1] Ministry for environment, spatial planning and energy; Environmental Agency of the Republic of Slovenia, Ljubljana, Slovenia

[2] University of Ljubljana, Faculty of electrical engineering, Ljubljana, Slovenia

Key words: commutator motor, brush-to-segment width ratio, commutation, motion equation in electrical coordinates, minimization of electromagnetic interference.

Abstract: Commutator motors are widely used in several consumer appliances especially in household apparatuses and electrical hand tools. Therefore they are economically very important for their numerous production. Furthermore, due to their high shaft speed they achieve high power per a unit of volume, and they are produced at relatively low costs for a unit of power. But these relatively powerful and economical machines are unfavourable electromagnetic interference sources, which can compose the disturbing electromagnetic environment. Electromagnetic interference is caused primarily by a commutation of a current in an armature coil. The paper describes a theoretical approach to a minimization of electromagnetic interference of the commutator motors through concepts of analytical mechanics. These concepts involve both a magnetic system and motion systems of the motor using the magnetic and the electric energy. Further, a criterion is established for an optimization of a brush-to-segment width ratio of a commutator-brush system due to minimal amplitudes of electromagnetic interference and minimal contents of harmonics in a spectrum of electromagnetic interference.

Zmanjšanje EMI kolektorskih motorjev z optimiranjem prekrivanja lamel komutatorja

Ključne besede: kolektorski motor, prekrivanje lamel, komutacija, gibalna enačba v električnih koordinatah, minimizacija elektromagnetne interference

Izvleček: Kolektorski motorji so množično uporabljeni v številnih napravah široke potrošnje, posebno v gospodinjskih aparatih in električnih ročnih orodjih. Zaradi njihove velikoserijske proizvodnje so ekonomsko zelo pomembni. Ker dosegajo velike hitrosti vrtenja, razvijejo veliko moč na enoto volumna in njihovi proizvodni stroški na enoto moči so relativno majhni. Toda ti zelo ugodni stroji glede na moč in ekonomiko so nezaželeni elektromagnetni interferenčni izvori in lahko tvorijo motilno elektromagnetno okolje. Elektromagnetne interferenčne motnje povzroča predvsem komutacija toka v svitkih armature. Članek teoretično obravnava minimiziranje elektromagnetnih motenj kolektorskih motorjev z uporabo analitične mehanike. Tak pristop zahteva obravnavo magnetnega in gibalnega sistema motorja v povezavi z magnetno in električno energijo. Izdelan je bil tudi kriterij za optimiranje prekrivanja lamel glede na minimalne amplitude elektromagnetnih motenj in minimalno vsebnost višjih harmonskih komponent v njihovem spektru.

1. Introduction

When researching the phenomenon of the commutation two methods based on the theory of electric circuits are used /1,2,3/. When these methods are used, each specific motor require its own model, valid only for this motor. The first method applies transfer differential equations to the electric circuit of the motor, which is built up by time-depending elements. The second method is also applied to the electric circuit of the time-depending elements, but it is based on a time-varying circuit topology.

In a general case, when we are looking for general rules of the commutation, it is more convenient to use the principles of analytical mechanics in systems of the motor applying the magnetic and the electric energy of the motor. The commutation is the process of reversing the current in one coil; it begins when a brush bridges two commutator bars which are the terminals of this coil, and finishes at the latest when the brush leaves out the leading commutator bar of this coil with possible transients; geometrical actions of bridging and leaving are defined as a geometrical performance of the commutation.

Due to rotation of the commutator the brush consecutively makes and breaks electric contacts over the armature coils. Making and breaking the electric contact over the armature coils are succeeding each other: making the electric contact over one armature coil is followed by breaking the electric contact over the other armature coil. This movement is defined in this paper as an interchange of positions of the armature coils under the brush bridge. An interval between two consecutive interchanges is a pause. Because the armature coils are magnetically coupled, the commutation of the current and geometrical displacements causes changes in the magnetic energy of the whole magnetic system of the motor. Due to these changes the voltage is induced in the windings of the motor, which is the source of electromagnetic interference.

A mathematical model of the magnetic system is built up with a purpose to calculate the magnetic energy and its changes due to the commutation of the current and the geometrical displacements of the armature coils. This mathematical model contains an algorithm of dynamic calculations of a magnetic reaction of the armature and also takes into account a kinetic behaviour of the commutator regarding the brushes. This algorithm is the only way to determine the changes in the magnetic energy as a function of a rotation angle.

The voltage of the source of electromagnetic interference is a result of a conversion of the magnetic energy into the electric energy of stray capacitance. Therefore motion systems in electrical and in geometrical coordinates are established. Because both of the motion systems are defined by conceptual the same motion equation, the experimental results of one system evaluates also the another system.

In the case without transients at the end of the commutation process, electromagnetic interference results from changes of the magnetic energy of the machine. This kind of emitted interference propagates mostly conductively to a supplying network. There are almost no radiated emissions.

Intending to make mathematical models more clear, a d.c. serial motor with two poles is chosen as a representative type of the commutator machines.

2. The magnetic system

To establish the mathematical model of the magnetic system /4/ of the DC motor, a system of Maxwell's equations in the integral form is to be solved. As the main purpose of the presented model is to study the commutation, it is particular important to detail the total current in relation with the geometry and kinetic behaviour of the motor.

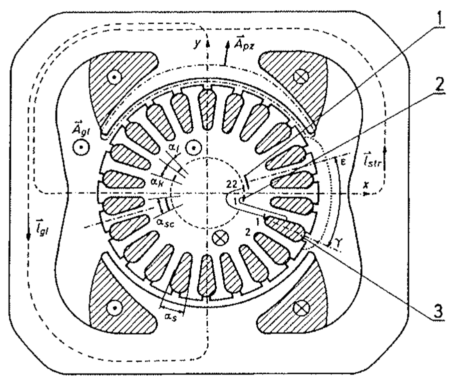

Fig.1: The schematic cross-section of the motor: (1) the brush, (2) the commutator bar, (3) the armature coil.

The total current is obtained by an integral of a conduction current density through a surface (A_{gl}) defined by the closed contour (l_{gl}) - Fig.1. A geometrical distribution of the coils around the armature is closely taken into account. Thus the total current depends on the number of turns of stator coils (N_{st}) and the number of turns of the armature coil (N_s). Furthermore it is function of a stator current (i_{st}), a coefficient of the coil ($\kappa(\alpha_s)<1$ and $\kappa(\alpha_s)\approx 1$), depending on a width of the coil (α_s), and also of the number of commutator segments (n_l), which equals the number of slots of the armature in this case. It also depends on an angle defining a position of the coils towards the commutator segments (γ) and on a rotation angle (α):

$$\iint_{A_{gl}} J\cdot dA = N_{st}\cdot i_{st} + \kappa(\alpha_s)\cdot N_s\cdot \left(\cos(\alpha+\gamma)\cdot\sum_{i=1}^{i=n_l} i_s\left(\alpha-(i-1)\cdot\frac{2\cdot\pi}{n_l}\right)\cdot\cos\left((i-1)\cdot\frac{2\cdot\pi}{n_l}\right)+ +\sin(\alpha+\gamma)\cdot\sum_{i=1}^{i=n_l} i_s\left(\alpha-(i-1)\cdot\frac{2\cdot\pi}{n_l}\right)\cdot\sin\left((i-1)\cdot\frac{2\cdot\pi}{n_l}\right)\right) \quad (1).$$

An expression $i_s(\alpha\text{-}(i\text{-}1)\ldots)$ at $i=1$ in equation (1) is a current distribution around a circumference of the armature as a function of the angle, and of intervals of the geometrical performance of the commutation. These intervals depend on widths of the commutator segment (α_k), of the commutator bar (α_l), of the brush (α_{sc}) and on an angle of the brushes towards a geometrical neutral point (ε) - Fig.1.

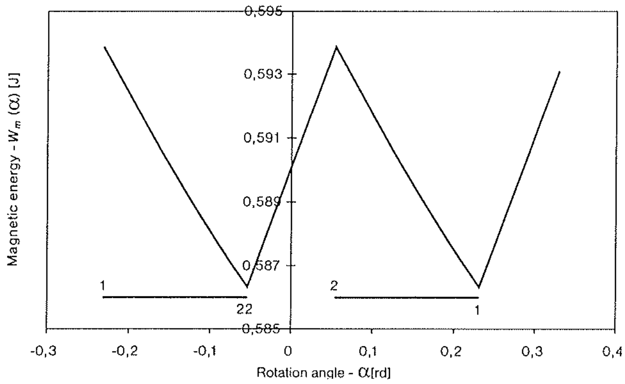

Fig.2: The magnetic energy $W_m(\alpha)$, depending on the rotation angle, at the linear commutation and the brush-to-segment width ratio of 1.8 .

Resolving the system of Maxwell's equations, the magnetic field intensity in all parts of the magnetic system is determined as a function of the rotation angle of the armature. Due to a magnetization curve of a ferromagnetic material, the magnetic energy (W_m) depending on the rotation angle is calculated.

Analysing the function of the magnetic energy $W_m(\alpha)$ in Fig.2, the two interchange intervals are represented by straight lines from the point *1* to the point *22* and from the point *2* to the point *1* with the pause between them. The geometrical performance during the commutation lasted from the first point *1* (the instant of making the electric contact over the coil 1) to the last point *1* (the instant of breaking the electric contact over the coil 1). It includes both interchanges.

3. The motion system in electrical coordinates

The motion equation is describing the relation between the kinetic and the potential energy of any motion system in general /5/. Instead of the kinetic and the potential energy, the magnetic (*Wm*) and the electric energy (W_e) are applied respectively in a particular system where the generalized coordinates are electric charges and the generalized velocities are currents. The system is subjected to actions of the conservative forces and the nonconservative forces, which result in the active and the reactive force. The active force is defined by the derivative of the magnetic energy with respect to a supplying current of the system and further on with respect to time. But the reactive force is defined by the derivative of Rayleigh's dissipation function (F_i) with respect to a current which causes dissipation. The dissipation function is one half of an electric power dissipation /6/.

The magnetic energy is obtained from the magnetic system of the motor. From the viewpoint of the electrical coordinates, it depends only on currents which are the current of the motor (i_m) and a current (i_c) through stray capacitance. On the other hand the electric energy depends only on the electric charge (q_c) found with stray capacitance of the motor. The time derivative of the electric charge is the current (i_c) through stray capacitance. The current through stray capacitance also causes the electric power dissipation. Taking into account isomorphism of time {*t*,+} and rotation angle {α,+}, the equation of motion is written as follows:

$$\omega_r \cdot \frac{d}{d\alpha}\frac{\partial W_m}{\partial i_c} + \frac{\partial W_e}{\partial q_c} = \omega_r \cdot \frac{d}{d\alpha}\frac{\partial W_m}{\partial i_m} - \frac{\partial F_i}{\partial i_c} \quad (2).$$

Summands of this equation (2) are generalized forces, which are, speaking in terms of the electrical coordinates, relevant voltages of the system. The first summand is the voltage across a conceptual inductance of the motor, caused by the current through stray capacitance. The second one is the voltage (u_c) across a conceptual stray capacitance, which is the voltage on terminals of the motor. On the right side, the first summand is the induced voltage (u_{ind}) due to the changes of the magnetic energy, which is, in fact, the voltage of the electromagnetic interference source. Further on, there is also the voltage drop across a conceptual resistance due to the electric power dissipation. Using equivalent concentrated elements instead of the conceptual ones, and introducing the voltage (u_c), the second-order differential equation (3) is obtained:

$$\omega_r^2 \cdot L_{eq} \cdot C_{eq} \cdot \frac{d^2 u_c}{d\alpha^2} + \omega_r \cdot R_{eq} \cdot C_{eq} \cdot \frac{d\, u_c}{d\alpha} + u_c = \omega_r \cdot \frac{d}{d\alpha}\frac{\partial W_m}{\partial i_m} \quad (3).$$

The induced voltage (u_{ind}) is derived from the magnetic energy, but the voltage on the terminals of the motor (u_c) is the result of the equation (3) - Fig.3. Further on, amplitudes and rates of the harmonics in the spectrum of the voltage u_{ind} (α) are to be determined as functions of the brush-to-segment width ratio. The minima of these functions are then established thus providing the basis for the optimization of the brush-to-segment width ratio.

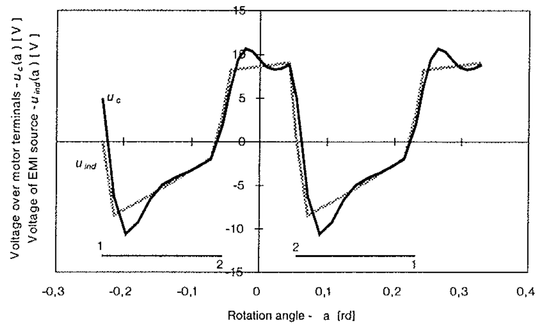

Fig.3: The voltage of EMI source u_{ind} (α) and the voltage over the motor terminals u_c (α), depending on the rotation angle, at the linear commutation, the brush-to-segment width ratio of 1.8 and the shaft speed of 7.8 kcys/min.

4. The motion system in geometrical coordinates

A torque of the motor is determined using the motion system of the motor in geometrical coordinates. To simplify the motion system, it is presumed, that the system works without mechanic losses. Consequently, the motion system applies, beside conservative forces, only two nonconservative forces, which is the torque of the motor and a torque determined by the electric dissipation. Due to isomorphism of the angles {α} and time {*t*}, the trivial value of the torque would result. Therefore displacements of the magnetic field of the armature, which directly cause the changes in the magnetic energy, are described by another angular coordinate (α^*). The rule of correspondence {α}→{α^*} is obtained from the mathematical model of the magnetic system. As an inverse rule of correspondence does not exist, the torque of the motor is related to the actual angular coordinate rotation (α) and therefore to time. The equation of motion in geometrical coordinates results in the following equation:

$$M(\alpha) = \frac{\partial \left(W_m^*(\alpha^*) - W_e^*(\alpha^*)\right)}{\partial \alpha^*} - \frac{\partial F_i(\omega_r)}{\partial \omega_r} \quad (4).$$

While optimizing the commutator-brush system, average values of the torque depending on the brush-to-segment width ratio are to be calculated.

5. The optimization of the brush-to-segment width ratio

To minimize electromagnetic interference, the brush-to-segment width ratio is chosen to be optimized. It is a quo-

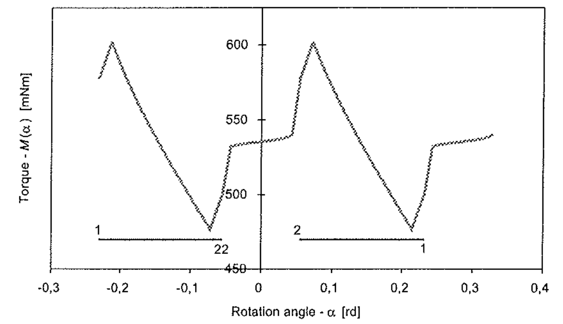

Fig.4: The torque $M(\alpha)$, depending on the rotation angle, at the linear commutation and the brush-to-segment width ratio of 1.8 .

tient of the width of the brush and the width of the commutator segment.

According to the mathematical models of the magnetic and the motion system in the electrical coordinates - both are supplemented with the variable brush-to-segment width ratio (r) - the functions $W_m(\alpha,r)$ and $\mathbf{u}_{ind}(\alpha,r)$ are established for a chosen domain of definition of the brush-to-segment width ratio.

A function of amplitudes $U_{ind}(r)$ of the voltage $u_{ind}(\alpha,r)$ and a function of the rates $h_{ind}(r)$ of the harmonics in the spectrum of this voltage, both depending on the brush-to-segment width ratio, are now obtained.

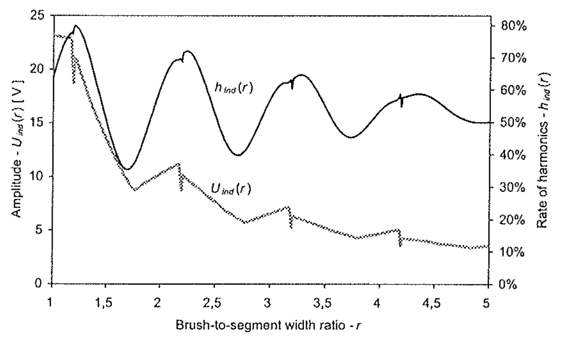

Fig.5: The amplitude of the EMI source voltage $U_{ind}(r)$ and the rate of the harmonics $h_{ind}(r)$, depending on the brush-to-segment width ratio, at the linear commutation and the shaft speed of 7.8 kcys/min.

The functions $U_{ind}(r)$ and $h_{ind}(r)$ in Fig.5 are set up for the linear commutation current. It is seen that the minima of the amplitude function $U_{ind}(r)$ and the minima of the harmonics rates function $h_{ind}(r)$ do not coincide. The optima of the brush-to-segment width ratio are achieved at the minima of the amplitude functions.

The function of the torque M(r) depending on the brush-to-segment width ratio is shown on a diagram in Fig.6.

The function monotonously decreases with the increasing values of the brush-to-segment width ratio. This function has no optima of the brush-to-segment width ratio.

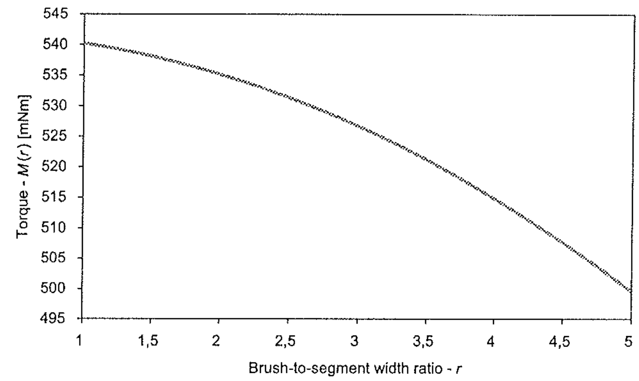

Fig.6: The torque $M(r)$, depending on the brush-to-segment width ratio, at the linear commutation.

6. The experimental evaluation of both motion systems

The numerical analyses, carried out previously by the mathematical model, took into account technical data of the commutator motor produced by Slovenian firm DOMEL. In the mathematical models, the linear commutation of the current was presumed up to this point. The motor under test, loaded by an electromagnetic brake, was especially assembled with measuring slip rings and brushes to measure currents through two successive armature coils. The measurement shows that the current of the armature coil reverses before the geometrical performance of the commutation is accomplished. There is overcommutation of the current. To compare the mathematical results and measured results, a simulation of overcommutation is done.

Diagrams in Fig.7 show the calculated induced voltage of the electromagnetic interference source and the measured voltage over the terminals of the motor.

There are differences between both functions, and these differences were also conceptually predicted by the diagrams in Fig.3. The simulated induced voltage is the source voltage, but the voltage over the terminals of the motor is the response of a spatial distributed electric circuitry of the motor on the induced voltage. The induced voltage cannot be measured. The electric circuitry of the motor has more than one resonant frequency, so the simulation of the response on the induced voltage by it is more complex than just solving the differential equation (3). Considering the optimization of the commutator-brush system to minimize electromagnetic interference, it is more appropriate to minimize the induced voltage for it is the source voltage. The comparison of both voltages, however, verifies the electric motion system of the motor.

The motion system in geometrical coordinates is established to determine the function of the torque of the motor depending on the brush-to-segment width ratio. There is also another purpose of the mathematical model of this mechanic motion system: verification of both motion systems for they are based on the same motion equation, but using the different kind of generalized coordinates.

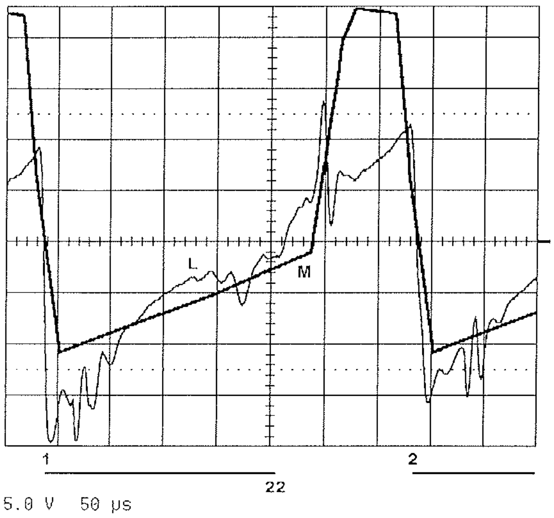

*Fig.7: The voltage of EMI source $u_{ind}(\alpha)$ simulated by the mathematical model (**M**) and the voltage over the motor terminals $u_c(\alpha)$ measured in the laboratory (**L**), depending on time at the shaft speed of 7.8 kcys/min.*

This verification is made by comparison of the calculated torque characteristic of the motor for d.c. conditions and two measured torque characteristics, one achieved by d.c. measurements in the laboratory, but the other by a.c. measurements in the factory - Fig.8. Both torque characteristics, simulated and measured in the laboratory are very close to each other. The torque characteristic, measured in the factory, shows lower values of the torque than the other two characteristics. There are some considerations about possible causes of the difference: the different kind of supply currents (d.c. and a.c.), tolerances of assembled parts and materials, some inadequacies in the simulation of the magnetic reaction of the armature and in the simulation of saturation of the ferromagnetic material.

Taking all comparisons of the simulated and the measured quantities into account, it can be considered that the magnetic and both motion systems and also the method of the optimization of the brush-to-segment width ratio are verified.

7. Conclusions

On the principles of analytical mechanics the mathematical model was established applying the magnetic and the electric energy of the commutator motor overall to optimize the commutator system of the motor. Knowing the current distribution around the circumference of the armature, the brush-to-segment width ratio can be optimized to minimize the amplitude of electromagnetic interference and/or the rate of the harmonics in its spectrum. In practice the arcing occurs between the brushes and the commutator bars, and in this case, the optimization method leads to minimal arc energy and so far to longer duration of the commutator.

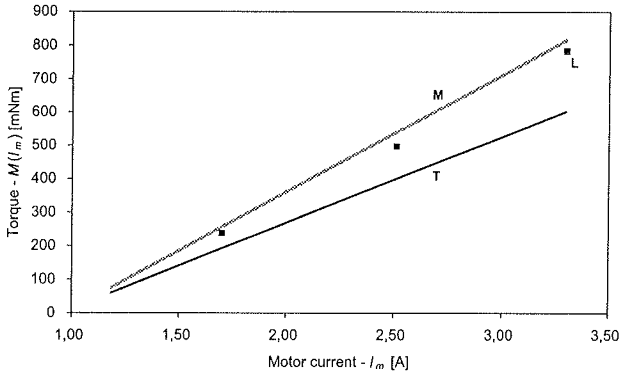

Fig.8: The torque characteristics of the motor under the test: obtained by the mathematical model (M) - d.c. conditions, measured in the laboratory (L) - d.c. supply, tested in the factory (T) - a.c. supply.

References

/1/ SACK, J.; SCHMEER, H.: 'Computer-aided analysis of the RFI voltage generation by small commutator motors', IEEE-EMC Society, 1985, Symposia Records 1955 to 1995, CD-ROM Database 1996.

/2/ SURIANO, J.; ONG, C.M.: 'Modeling of electromechanical and electromagnetic disturbances in DC motors', IEEE-EMC Society, 1989, Symposia Records 1955 to 1995, CD-ROM Database 1996.

/3/ SURIANO, C.R.; SURIANO, J.R.; THIELE, G.; HOLMES T.W.: 'Prediction of Radiated Emissions From DC Motors', IEEE-EMC Society, 1998, IEEE Inc, Symposia Records 1996 to 1999, CD-ROM Database 1999.

/4/ WOODSON, H.H.; MELCHER, J.R.: 'Electromechanical dynamics, Part I: Discrete systems' (John Wiley & sons, inc. New York, London, Sydney 1968).

/5/ YAVORSKY, B.; DETLAF, A.: 'Handbook of Physics' (Mir publishers, Moscow 1975, English translation).

/6/ RAYLEIGH, J.W. STRUTT: 'The theory of sound' (Dover Publications, New York, 1945, Volume 1, 2nd edition).

dr. France Pavlovčič, univ.dipl.ing.
Ministrstvo za okolje in prostor, Agencija RS za okolje, Vojkova 1b, Ljubljana
tel.: +386 (01) 4784 098, fax: +386 (0)1 4784 054, E-mail: france.pavlovcic@gov.si

prof.dr. Janez Nastran, univ.dipl.ing.
Univerza v Ljubljani, Fakulteta za elektrotehniko, Tržaška 25, Ljubljana
tel.: +386 (01) 4768 282, fax: +386 (0)1 4264 647, E-mail: janez.nastran@fe.uni-lj.si

Prispelo (Arrived): 30.05.2002 Sprejeto (Accepted): 28.06.2002